Savoir vs connaitre in Chinese and some thoughts on lingenierie didactique

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Abstract: *The French near synonyms* "savoir" *and* "connaître" *cannot be distinguished in English, neither can the Chinese characters* zhi (知) *and* shì (識)*. The author would like to discuss some mathematics educational thoughts developed recently in Taiwan with a French school of mathematics educators who have established, among other achievements,* l'ingénierie didactique *since 1980s. The Taiwanese ideas originated in 2013 in response to the needs of a guideline for the design of the literacy-oriented mathematics curriculum for the newly implemented 12-year compulsory education. Our key ideas are* (1) *think of mathematics as a language,* (2) *distinguish* zhi *and* shì *and the later one is crucial for literacy, and* (3) *the work and responsibility of an educator is more like an engineer than a scientist. Five years later, we realized that our thoughts might have been very similar to those well established by the Didactique scholars. Therefore, here we are to present our perspectives and to learn from the eminent colleagues.*

ZHI, XÍNG, AND SHÌ IN TAIWAN

In order to lay out a guideline for the design of the literacy-oriented mathematics curriculum for the newly implemented 12-year compulsory education, Fou-Lai Lin led a small team of researchers to conduct a pilot study. It is in the report of that study the distinction of two notions of *knowing* is proposed (Lin, Shann, Lee, & Cheng, 2013). The Chinese word for *knowledge* is composed by two characters zhi-shì (知識), each of them can be used both as a verb and as a noun. The two notions of knowing are identified by *zhi* and *shì*. The distinction between these two notions are well perceived by classical Chinese intellectuals, as they were actually used individually in Chinese classics. Although they are seldom used separately in contemporary speaking and writings, their distinct meanings are still in the culture. It is not hard for a Chinese scholar to perceive that *zhi* is about recognizing someone or something as he/she/it is while *shì* is about knowing why someone is who he/she is or why something is what it is. We knew that there are no

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corresponding words in English, and we proposed to approximate *zhi* as "to know" and *shì* as "to understand", "to make sense of", "be aware of", or "have an insight into" (Shann, 2016).

For instance, Taiwanese middle school students all know that equations have their corresponding graphs, but they usually do not understand how equations are associated with graphs. A typical textbook will have a few sentences about René Descartes, follow by one (no more than two) activity that draw four, five, or six points with integer coordinates on the plane, then race to the conclusion that the corresponding graph is a straight line. The situation is made worse in Taiwan as a large portion of students attend cram schools or hire private tutors, so they know how to draw lines or parabolas skillfully before schoolteachers arrive the same topic. As a result, many middle school students cannot ``memorize'' for a long time that the intersections of a parabola with x-axis give information about solutions of a quadratic equation.

As the curriculum was officially announced by Ministry of Education in 2018, we suggested textbook authors, teachers, and colleagues to refer to the three-facet construct for their works, namely, to know, to do, and to *shì* (Shann, 2018). It means that when one designs a teaching material or a learning activity, he/she is suggested to keep in mind that there are three types of goals to take care of, namely:

- Zhi: To know the contents.
- Xíng (行): Be able to carry out the process, to perform typical applications.
- Shì: To understand why the contents or processes are worth to learn, to make sense of the learning, to be aware of the situations that might be suitable to call for the particular mathematics.

We urge colleagues in Taiwan to think of mathematics as a language, therefore the math-literacy is *literally* a literacy: to read and write mathematics properly (and hopefully proficiently). The curriculum was re-designed with this central thesis in mind, and the textbooks, teaching materials, assessments and mindsets of teachers are called to adjust accordingly. In the meantime, we also made sure to the teachers that they were mostly doing fine. The changes are not meant to be revolutionary; they are adjustments basing on the *status quo*. By referring to the three-facet curriculum construct, we assured teachers in Taiwan that they have done a good job in the first two parts, namely "to know" and "to do", which took up more than two-thirds of our responsibilities. Teachers and colleagues are suggested to pay more attention to the third part, to *shì*. Workshops are held around Taiwan to spread the message, to explain the construct, to show examples, and to exchange experiences.

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For instance, to help connecting an equation with its graph, teachers are suggested to design activities that provide students a chance to realize that the graph is a collection of (many many) points with coordinate values ``satisfying'' the equation. One may start with integer x values like -2, -1, 0, 1, 2 at the first run, then the halves like $-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$ for the second run, then the quarters like $-\frac{7}{4}, -\frac{5}{4}, -\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}$ for the third run, and so on (with calculators). Allow the image of a graph emerge gradually. In order to engage those students who think they know the ``fast way'' of getting the right answer, teachers can use unusual equations. The challenge of graphing $y=\frac{1}{x}$ might be good for grade 7, $4x^2+y^2=4$ for grade 8, and $x^3+2xy+y^2=0$ for grades 9 and 10.

SAVOIR, CONNAÎTRE, AND ADIDACTICAL SITUATIONS

I have to concede that math educators in Taiwan are less aware of the activities in continental Europe, with probably an exception for Finland in recent years. So I had a mixed feeling of blessed and ashamed when I realized in 2018 that a school of eminent scholars associated with *l'Association pour la Recherche en Didactique des Mathématiques* (ARDM) have been investigating the analogous ideas more than 30 years ago. The most intriguing ideas are the conceptions of *savoir* vs *connaître*, of *didactical* vs *adidactical* situations, and of *didactical engineering* (DE) as a research and design methodology.

To my understanding, *savoir* and *connaître* respectively correspond to *zhi* and *shì* perfectly. My judgement was made basing on the following examples. The next quotation on the distinction between *savoir* and *connaître* in the context of mathematics teaching and learning, or *Didactique*, is coincide with ours, only in a much professional tongue.

> The process [of institutionalization], which connects knowledge in situation (*connaîssance*) and institutional knowledge (*savoir*), works in both directions. During the construction of knowledge, the initial usefulness in situations is gradually forgotten. Knowledge is formalized, which is very important in order to create a coherent body of knowledge, known as mathematics. However, if you need to use mathematics to solve a problem, you have to understand its usefulness in situations, which is very different from understanding formal mathematics. Thus there is a dialectical link between formalized knowledge (*savoir*) and knowledge in situation (*connaîssance*). (Margolinas & Drijvers, 2015, p.899)

For the purpose of explaining the notion of *shì* and the rationale to place it on the curriculum construct, Chevallard provided us the most concise expression: To teach the facet of *shì* of a

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mathematical topic is to make students realize "*what its reasons are to be here, in front of us, waiting to be studied, mastered, and rightly utilised for the purpose it was created to serve.*" (Artigue, 2009, p.14).

French and Chinese are different languages/cultures after all, so there should be some differences between these two pairs of words for knowing. Indeed, although *savoir* and *connaître* are two words, a casual treatment of their distinction "*would be a serious error*" (Brousseau, Brousseau, & Warfield, 2004, p. 3). Here is the reason:

> "*Connaître*" a theorem means to have bumped into it sufficiently often to have an idea of its context and uses and of more or less how it is stated; "*savoir*" a theorem means to know its statement precisely, how to apply it, and probably also its proof. On the other hand, when it comes to an entire theory, with a collection of theorems and motivations and connections, what is required is to *Connaître* it. *Savoir* at that level is not an available option. … Isolated parts are acquired as *savoirs* connected by *connaissances*. Without the *connaissances*, the *savoirs* have no context and are swiftly mixed or lost. Without the *savoirs*, the *connaissances* are more touristic than useful. (Brousseau *et al.*, 2004, p.2)

Chinese is better off on this aspect of pragmatics, as *zhi* and *shì* are spoken in conjunction in our daily life to express the meaning of "knowledge". It is natural for a Chinese to understand the roles of mutual support of *zhi* and *shì*, and to accept the ideal of weaving "to know", "to do", and "to *shì*" for the entire fabric of the learning.

In the last part of this section, I would like to discuss the adidactical situation. It is argued to be the appropriate way of teaching knowledge in situation. Do we in Taiwan agree that it is adequate for *shì* in the *Didactique*? I am afraid not, if adidactical situation is what the following quotation means:

> The student knows very well that the problem [posed by teacher] was chosen to help her acquire a new piece of knowledge, but she must also know that this knowledge is entirely justified by the internal logic of the situation and that she can construct it without appealing to didactical reasoning. Not only can she do it, but she must do it because she will have truly acquired this knowledge only when she is able to put it to use by herself in situations which she will come across outside any teaching context and in the absence of any intentional direction. Such a situation is called an *adidactical situation*. (Brousseau, 2002, p.30)

Although the statements are mostly undeniable, it is too *radical constructivist* for us. We believe in the efficacy of teaching by students' activities in situations, yet Taiwanese didactical culture values so much on the efficiency and it is hard to engage the students for devolved responsibilities in the sense of the *didactic contract*. Our considerations on situations for *Didactique* is to take

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language acquiring as an analogy. We think mathematics beyond grade 4 should mostly be considered a foreign language (or L2). Just like L2-learning can be done by sheer translation of the mother language (L1) or by an immersion of cultural situations, mathematics can be learned by well-organized instructions or by well-designed activities. The former is usually suitable for learning formalized knowledge (the facet of *zhì*) and the later for knowledge in situation (the facet of *shì*). Therefore, if we want to implement *adidactical situations* in Taiwan, it is most likely to success when it is thought of a scaffold like the L2-learning supported by L1. It can take the forms of controlled experiments, games, or stories told by teachers.

We agree that "*each item of knowledge can be characterized by a (or some) adidactical situation(s) which preserve(s) meaning; we shall call this a fundamental situation.*" (Brousseau, 2002, p.30). For each mathematical topic in the curriculum, we try to suggest a fundamental situation for the purpose of teaching its facet of *shì*.

We would like to add one more example by the end of this section. To provide one more chance for understanding the graphs of equations, and to prepare students for the insight of tangent lines at the same time, we would invite students to draw local graphs. For instance, sketch the graph of unit circle $x^2 + y^2 = 1$ inside a square with sides 0.02 centered at the point (0.6, 0.8). This is a ``weird'' task for students with their first encounter. When they finally struggle their way out, they realize surprisingly that a circle looks like a straight line when it is looked close enough. They can read the slope of that line: $-\frac{4}{3}$ which is the slope of the line tangent to the circle at that point. With a solid understanding of the meaning of graphs of equations, and with the help the graphing devices (or apps), students are ready to explore the global and local phenomena of the graphs of polynomial functions. One of the expected discoveries is that all graphs of polynomial functions are straight lines when they are looked close enough. Analogous to the experience of tangent lines to circles, here comes the motivation that those lines are called tangent lines for polynomial functions.

DIDACTICAL ENGINEERING

We couldn't agree more that the work and responsibility of a teacher/educator is more like an engineer than a scientist. The four-phase methodology of DE—preliminary analyses, conception and *a priori* analysis, realization, observation and data collection, *a posteriori* analysis and validation—is not only practical but also beautiful (Artigue, 2015). It should be an insistence that engineering, including DE, must base on established knowledge (established by certain commonly accepted paradigm), with an emphasis on epistemological and mathematical sensitivity for DE in

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particular. However, we are not sure why the base knowledge has to be scientific, be it theory of didactical situations (TDS), anthropological theory of didactic (ATD), or any other paradigm which claim itself scientific.

To our understanding, one of the problems challenged DE is about reproduction (Artigue, 2009). Isn't it true that the whole point of thinking of *Didactique* as an engineering rather than a science, and to validate the conclusions by internal processes, is to avoid the intrinsic inadequacy of repetition in educational research? If scientific knowledge must build on certain kind of repetition, while *Didactique* is practically impossible to repeat, isn't it a contradiction to wish for a scientific status for *Didactique*? Instead, shall we claim our paradigm with the engineering approach which bases on established knowledge, including humanities (and not excluding sciences)? For example, the argumentation that there are two notions of knowing, and the *Didactique* is an interplay of both of them, is not a scientific knowledge in any sense. However, with debate and critical commentary, it might eventually be accepted as a knowledge. Or, shall we say that it might eventually be accepted as an *axiom*? Would a set of axioms suits *Didactique* more than any fundamental knowledge?

By *Didactique Axioms* I mean not only the assumptions we made for the school math contents, but also the assumptions we take for curriculum design. For the first kind of axioms, the Fundamental Assumption of School Mathematics (FASM) proposed by H. Wu (2011, p.376) is a typical one. I would like to add another candidate: the graph of two-variable linear equations are straight lines. Students usually learn this topic before they have enough of prerequisites in geometry to justify the conclusion. Although students can usually accept the fact by practical experience, it might be better off giving a legal mathematical support. An example for the second kind of axioms is the assumption that the developments of mathematics concepts of an individual is proportional to that in the history. Some may acquire the concepts faster than others, but they all follow the same sequence and do not reverse the order. For instance, the sense of uncertainty might appear before the ability to manipulate symbolic unknowns. However, according to history, to handle uncertainties by numbers (probability) shall come after basic algebra. It is clear that the introduction of cartesian coordinates and line equations before enough of plane geometry violates the foregoing axiom. We choose to do it for the sake of efficiency in curriculum design. Therefore, we must be aware of the potential risk and pay much more attention to this topic.

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