

On the Versatility of a Math Major

多芸な数学系の学生になるために



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<http://libai.math.ncu.edu.tw/~shann/>

Think of Mathematics as a Language

数学を言語として認識すれば

Math is Built-in



1, 2, 3, 4, ..., 100, ...

余り分

不足分

合わせる分

分ける分

God Creates Natural Numbers



示神

But why on earth ...



foreign language

外語

alien language

宇宙人語

We all Speak and Write



How many can write 俳句

秋近き 心の寄るや 四畳半

How many can create stories

小川洋子 《博士の愛した数式》

Many Layers of

Language Acquisition
and Mastery

Math Professors 俳人



Prepare Yourself



No individual curriculum.

Up to you

what to grasp

how much to grasp

to prepare yourself

as versatile as possible

Math Education: Practice and Philosophy

数学教育の実践と理念

Native Languages



- Pick up in an immersion of a realistic environment
- Learn by examples
- Postpone the divorce of Math and Native Language

Math Edu Fundamentals



- Learn in an immersion environment
- Teach by examples

Man-Made Universe



- Mathematics
- Computer Games
- Board Games
- Card Games

Roles in a Game



- Compatibility
- Inherit old rules
- Interact with old roles
- New rules and Exceptions

Example: 負数

Whole Numbers

Negatives

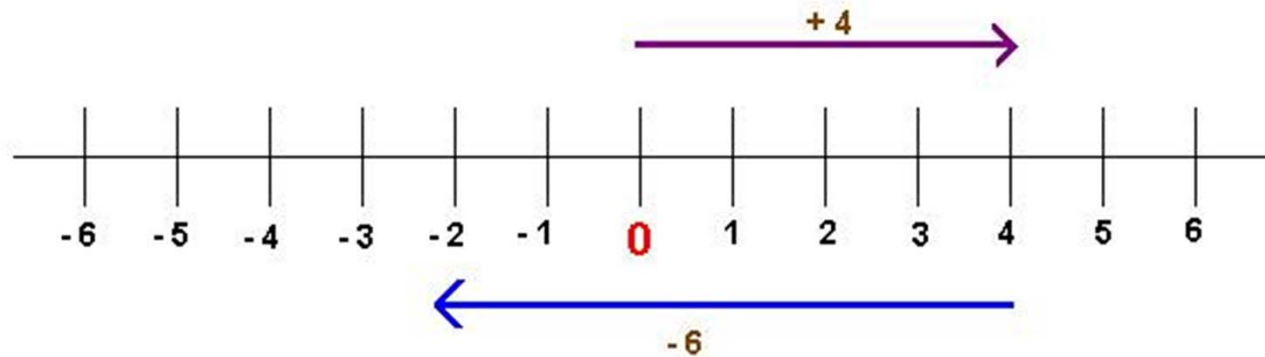
Integers



Man Creates Other Numbers



We create negative numbers
just like we create card games



$$3 + (-1)$$

$$5 + (-3)$$

$$4 + (-6)$$

$$(-2) + (-1)$$

三人成虎



No matter how absurd an idea is, three examples are enough for you to believe it.

People can usually derive a general rule by very few examples.

攻略：Fall Back



There are 8 cases of

$$a \pm b, \quad \text{for } a, b \in \mathbb{Z}$$

Fall Back to only Two:

$$m + n$$

$$m - n, \quad \text{where } m \geq n$$

$$\text{for } m, n \in \mathbb{N}$$

What 負数 are for?



- Directional measurement with a reference center, e.g. temperature and financing.
- Eliminate subtractions.

Learn Math as a Language



- Chances are
ノーベル文学賞
フィールド賞
- To read, to speak, and to reason with the common sense.

Math Teacher as a Profession

職業としての数学教師

Non-Professions



- To work well in 大学入試
test problems**X**
- To love and care your
students **X**

知而不言



- Don't say it just because you know it
- Don't do it simply because it is doable
- Don't prove when there is no doubt

時而言

- Not only what to say but also when to say
- Need to know the

Context



Context



- What comes before and what goes after
- What students already know before the lecture

Go with a Purpose



- High school math is not the purpose in itself
- To make decision with Purpose in mind
- **Must:** $(15 - (-6)) \div (-3)$
OK: $-|x| \leq x \leq |x|$

From a Higher Point of View



- **Must:** $7 \div (-2) = 7 \times (-\frac{1}{2})$

No: remainder of $7 \div (-2)$

- **Need:** solve $ax^2 + bx + c = 0$

Must: 平方完成

Ok: $6x^2 + 5x - 6 = (3x - 2)(2x + 3)$

- $\int \frac{a_m x^m + a_{m-1} x^{m-1} + \dots + a_0}{x^n + b_{n-1} x^{n-1} + \dots + b_0} dx$ 部分分数

Cultural Contexts

文化的脈絡

Greek Paradigm

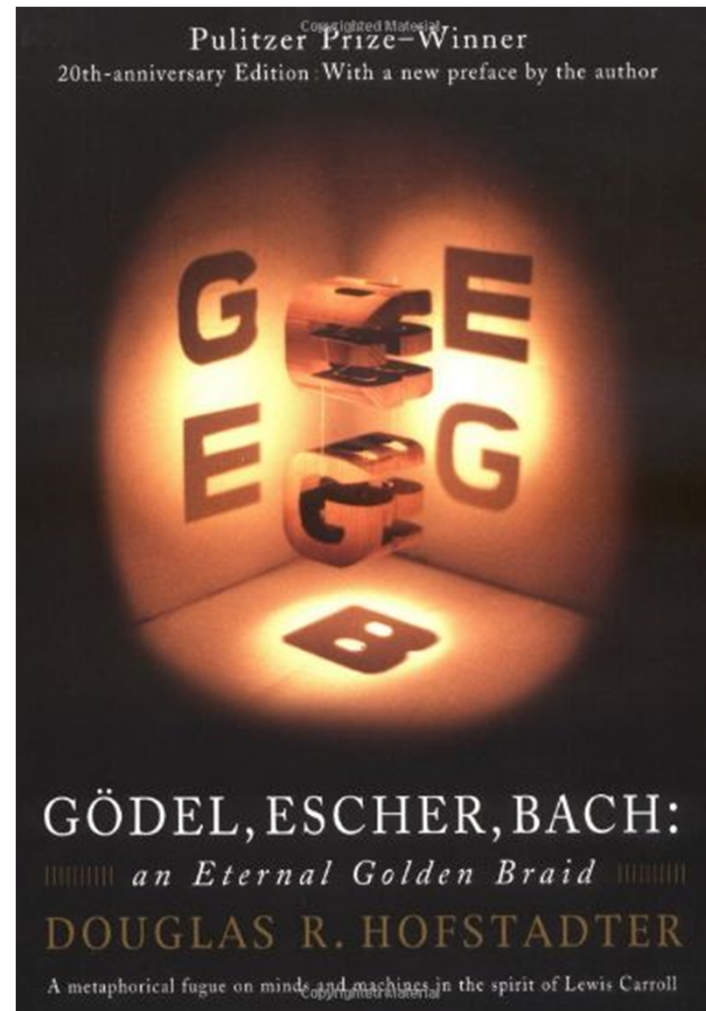
- 九章算經 和算
- Unique Math Style
- Western Culture
in
Greek Tradition



English References



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**MATHEMATICS IN
WESTERN CULTURE**
MORRIS KLINE



Japan Emerging



Life-Long Learning

生涯學習

Prepare in school



What are about to facilitate the future learning?

- Language
- Language
- Language

Natural Languages



- 國語
- English
英文卒業標準
- 華語
- Korean、Thai ...

Chinese Program at NCU



A Formal Languages



for relations and changes,
forms and shapes, quantities
and the measurement systems
and their manipulations, and
the description of data and
uncertainties.

Mathematics



Learn to be a fluent and efficient reader and speaker, not necessarily as a writer (俳人), though you may become one.

Languages for Automation

コンピューター言語

Math Text Processing



- **TEX** and **LATEX**

```
\documentclass[12pt,a4paper]{article}
\begin{document}
```

If the following limit exists:

\$\$

```
\lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}
```

\$\$

The limit is called {\it the derivative of $f(x)$ at x_0 }, denoted by $f'(x)$.

```
\end{document}
```

$$\lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Web Publishing



HTML: HyperText Markup Language

<HTML>

<BODY>

<H1>My First HTML 文件</H1>

<P>

Hello, world. How are you doing today?

I am learning HTML now.

</P>

<H2>第一節</H2>

<P>

其實 HTML 裡面中文也會通。

</P>

</BODY>

</HTML>



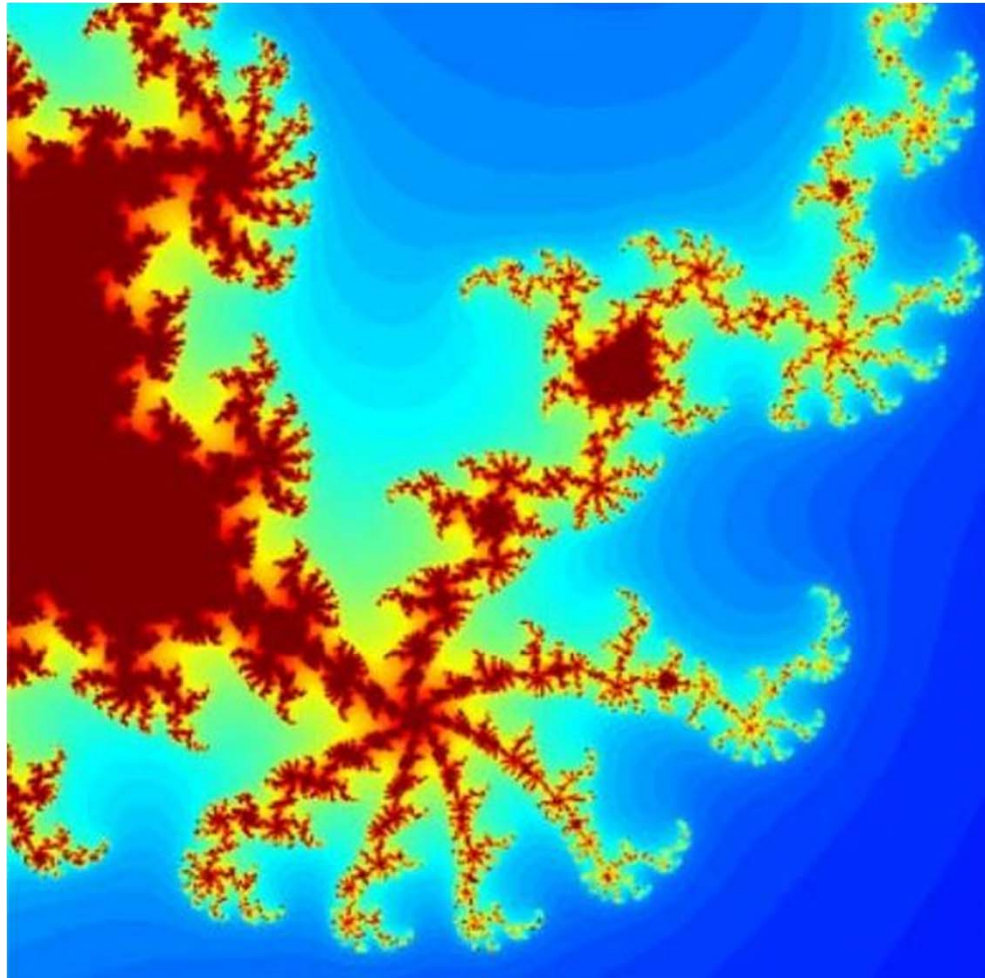
Relational Database



- SQL /sequel/ Structured Query Language
- JOIN: 集合の直積

```
select name, zip, zone, area, addressno  
from list join address join ZIP  
where list.addressid = address.id and  
address.zipid = ZIP.id  
and status = 'S' and gender = 'M'  
and (ZIP<300 or ZIP>700);
```


Matlab



```
s=400;          %% Matlab Script
initialx=-1.6810781;
initialy=-0.003132971;
width=0.01355;
threshold=100;
color=64;
x=ones(s,1)*linspace(initialx,initialx+width,s);
y=(linspace(initialy+width,initialy,s))*ones(1,s);
z=x+y*i;
out=zeros(s,s);
for u=1:s
    for v=1:s
        k=z(u,v);
        c=k;
        n=0;
        while (abs(k)>threshold & n<color)
            k=k^2+c;
            n=n+1;
        end
        out(u,v)=n;
    end
end
image(out)
axis xy equal off    %% 蔡少懷
```

Maple



- CAS: Computer Algebra System

```
> 1/2;  

$$\frac{1}{2}$$
  
> factor(6*x^2+5*x-6);  

$$(2x+3)(3x-2)$$
  
> ifactor(20141028);  

$$(2)^2 (3)^3 (43) (4337)$$
  
> evalf(Pi, 100);  
3.141592653589793238462643383279\  
50288419716939937510582097494459\  
23078164062862089986280348253421\  
17068
```

```
> Int((x^4+1)/(6*x^2+5*x-6), x);  

$$\int \frac{x^4 + 1}{6x^2 + 5x - 6} dx$$
  
> value(%);  

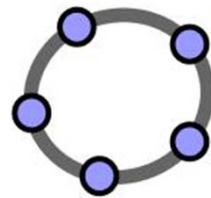
$$\frac{x^3}{18} - \frac{5x^2}{72} + \frac{61x}{216} - \frac{97}{208} \ln(2x+3) + \frac{97}{1053} \ln(3x-2)$$

```

Pedagogical Math Software



- Geogebra: High School Geometry and Algebra



GeoGebra

Software Dinámico de Matemáticas

- GSP: Geometers' Sketch Pad

THE GEOMETER'S

SKETCHPAD®



to C or not to C



- FORTRAN for scientific computing
- C is the common core
- Python ... maybe

Not to Waste your Life



We mathematicians have a trained talent to distinguish the hype from the core.

ALL are Numbers

すべてはナンバー

Basic Concepts of Computers



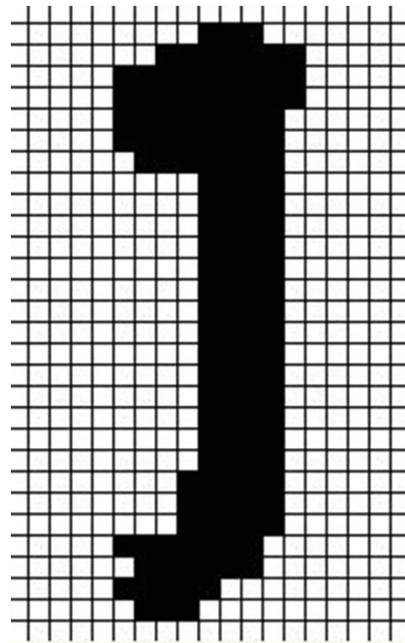
- Characters: CJK (中日韓), Unicode

	4E0	4E2	4E1	4E3
0	一 G 0-5027 J 0-1676 K 0-7673 B M40 C 1-4421 A 213021	北 G 5-1603 J K B C B-2262 A	丐 G 0-5604 J 0-4802 K B M42 C 1-4461 A 213029	丰 G 0-2365 J 1-1613 K 1-6612 B M45 C 1-4464 A 275958
1	丁 G 0-2201 J 0-3590 K 0-7943 B M42 C 1-4423 A 213022	両 G J 0-4630 K B C B-2261 A 333323	丑 G 0-1983 J 0-1715 K 0-8568 B M41 C 1-4460 A 275030	卯 G 3-1617 J 0-4805 K 1-5762 B C963 C 2-2143 A 216428
2	丐 G 5-1601 J 1-1601 K B C A	丢 G 0-2210 J K B C B-2263 A	刃 G J 1-1605 K B C A	串 G 0-2014 J 0-2290 K 0-4590 B A62A C 1-486B A 213035
3	七 G 0-3863 J 0-2823 K 0-8650 B M43 C 1-4424 A 213023	邝 G J 1-1607 K B C A	专 G 0-5508 J K B C A 27383F	弗 G 3-1608 J K B C E81 C 2-2531 A
4	丄 G J 1-1602 K B C B-2126 A	两 G 0-3329 J 1-1608 K B C B-243F A 273323	且 G 0-3950 J 0-1978 K 0-8306 B A542 C 1-4562 A 21302B	临 G 0-3357 J K B C A 275425
5	下 G J 1-1603 K B C B-2125 A	严 G 0-4947 J K B C A 27375A	丕 G 0-5607 J 0-4803 K 0-6164 B A541 C 1-4561 A 21302E	举 G J 1-1614 K B C A
6	厂 G J K B C A	並 G 8-1286 J 0-4234 K 1-6582 B ABC3 C 1-4864 A 213032	世 G 0-4232 J 0-3204 K 0-6506 B A540 C 1-4560 A 21302D	、 G 0-5628 J 0-4806 K B C B-2122 A 216431
7	万 G 0-4582 J 0-4392 K 0-5618 B C945 C 2-2126 A 274422	丧 G 0-4105 J K B C A 273663	卍 G J 0-5034 K B C A 2D302D	丷 G J K B C A
8	丈 G 0-5341 J 0-3070 K 0-7759 B M56 C 1-4437 A 213027	丨 G 0-5613 J 1-1609 K B C B-2121 A 216429	丘 G 0-3580 J 0-2154 K 0-4688 B A543 C 1-4563 A 21302F	丸 G 0-4572 J 0-2061 K 0-9215 B M59 C 1-443A A 213037
9	三 G 0-4093 J 0-2716 K 0-6318 B M54 C 1-4435 A 213024	凵 G 5-1628 J K B C A	丙 G 0-1791 J 0-4226 K 0-6016 B MFE C 1-455F A 21302C	丹 G 0-2104 J 0-3516 K 0-5101 B M46 C 1-4465 A 213038
A	上 G 0-4147 J 0-3069 K 0-6330 B M57 C 1-4438 A 213026	个 G 0-2486 J 0-4804 K B C B-212C A 273224	业 G 0-5021 J K B C A 27452D	为 G 0-4610 J K B C A 274951
B	下 G 0-4734 J 0-1828 K 0-8927 B M55 C 1-4436 A 213025	丫 G 0-4930 J 1-1610 K 1-7002 B M58 C 1-4439 A 283622	丛 G 0-2052 J K B C A 27352B	主 G 0-5487 J 0-2871 K 0-8111 B A544 C 1-4564 A 213039
C	丌 G 0-5602 J 1-1604 K B C946 C 2-2127 A 2D332A	丩 G 0-6760 J K B C A 4C5541	东 G 0-2211 J K B C A 274426	井 G J 0-4807 K B C964 C 2-2144 A 2D3053
D	丌 G 0-1827 J 0-4152 K 0-6084 B M43 C 1-4462 A 21302A	中 G 0-5448 J 0-3570 K 0-8173 B M4M C 1-4463 A 213034	丝 G 0-4331 J K B C A 27513B	丽 G 0-3286 J K B C E-2740 A 276256
E	与 G 0-5175 J 0-4531 K B C94F C 2-212F A 275432	卩 G J 1-1611 K B C950 C 2-2130 A	丞 G 0-5609 J 0-3071 K 0-6710 B A5E0 C 1-4722 A 213031	举 G 0-3057 J K B C A 275434
F	丐 G 3-1601 J K B C94D C 2-212D A 216424	丰 G J 1-1612 K B C A	丢 G J 1-1606 K 1-7742 B A5E1 C 1-4723 A 213030	丿 G 0-5615 J 0-4808 K B C B-2123 A 216433

Basic Concepts of Computers



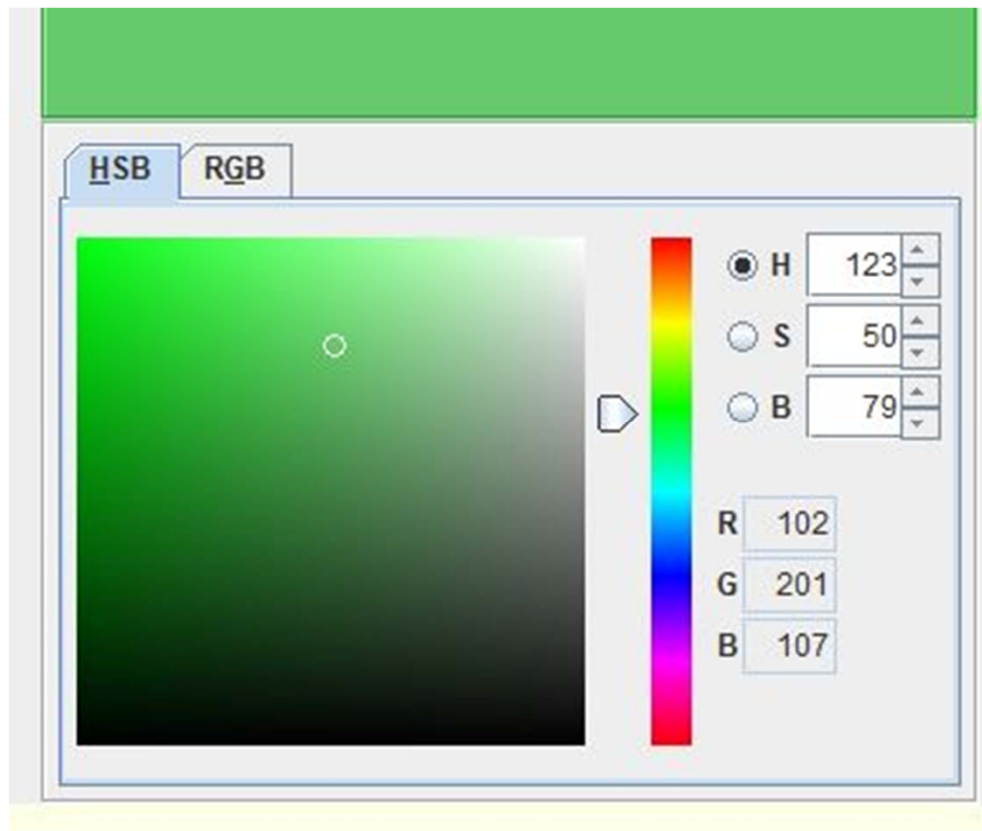
- Glyphs: Bitmap or Splines



Basic Concepts of Computers



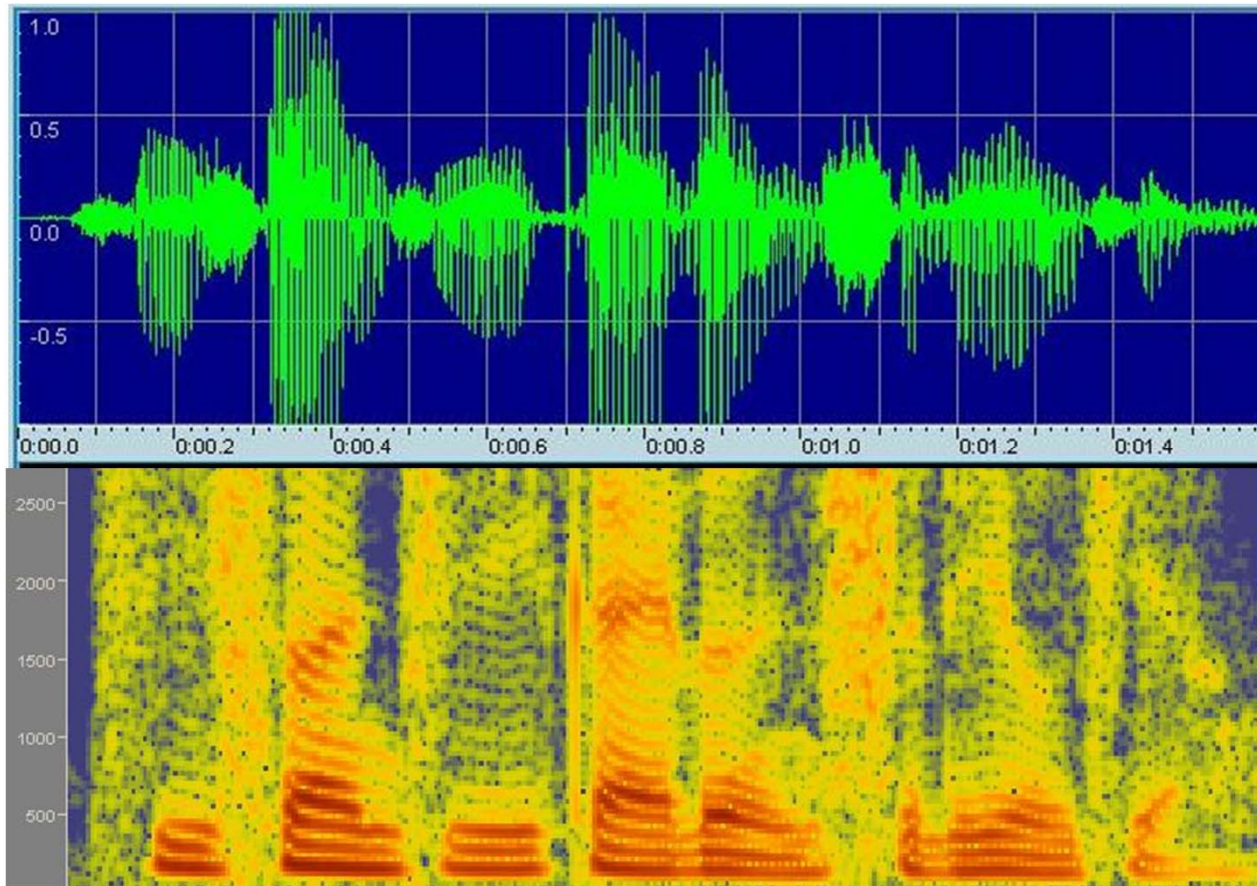
- Colors: Cylindrical or Cartesian Coordinate System



Basic Concepts of Computers



- Audio: Sequence of Numbers



Math Curriculum

数学カリキュラム

Four Corners of Foundation



Basic Concepts of Computers

微積分

統計

線形代数

微積分



- Math for the Continuum 連続体
- Super calculator for the past 300 years
- Many realistic objects are Discrete
- Calculus helped the invention of Computers

Discrete Math Models



- Math Tools for Discrete Models
- 離散数学
- 有限数学
- Algorithms アルゴリズム

Linear Algebra



- Some matrices are essentially the same
- Pure: identify the invariants
- Applied: conversions

Epilogue

結語



The End 

Thank you for your attention

<http://www.ncu.edu.tw>